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1 SEM TDC BOTH (CBCS) C 2

2021

(Held in January/February, 2022)

BOTANY

(Core)

Paper : C–2

(Biomolecules and Cell Biology)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Fill in the blanks : 1+1+1=3

(i) Conjugated protein consists of amino acids and a _____.

(ii) The hydrolysis of fat by organisms is called _____.

(iii) The cells of the multicellular organisms are held together by _____.

(b) Choose the correct answer : 1+1=2

(i) Non-reducing commercial sugar is glucose/sucrose/fructose.

(ii) Active transport in biological membrane utilizes sucrose/ATP energy/AMP/amino acid.

2. Write short notes on any *three* of the following : 4×3=12

(a) Enzymes as biocatalyst

(b) Function of messenger RNA

(c) Lipid synthesis in ER

(d) Role of microtubules

3. Describe the structure and functions of protein with example. 6+6=12

Or

Write short notes on the following : 4+4+4=12

(a) Ramachandran plot

(b) Functions of fatty acids

(c) Significance of meiosis

4. Why are mitochondria and chloroplasts considered as semiautonomous cell organallae? Describe in detail the structure and functions of mitochondria. 4+8=12

Or

Write explanatory notes on the following : 6+6=12

(a) Endosymbiotic theory

(b) Cell cycle

5. Write short notes on any *three* of the following : 4×3=12

(a) Function of Golgi apparatus

(b) Laws of thermodynamics

(c) Role of enzymes

(d) Phosphoglycerides

(e) Properties of water

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Total No. of Printed Pages—3

1 SEM TDC BOTH (CBCS) C 1

2021

(Held in January/February, 2022)

BOTANY

(Core)

Paper : C–1

(Microbiology and Phycology)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Choose the correct answer of the following : 1×3=3

(i) The thallus of Volvox is called as coenocyte / coenobium / colony / filament.

(ii) The principal pigment in Phaeophyceae is phycoerythrin / fucoxanthin / xanthophyll / phycocyanin.

(iii) Fertilization in Chlamydomonas is mesogamous / anisogamous / oogamous / isogamous.

(*b*) Fill in the blanks of the following : 1×2=2

(*i*) Many bacteria bear minute hairy structures on their cell wall, these are called _____.

(*ii*) Conjugation of bacteria was discovered by _____.

2. Write short notes on the following (any *three*) : 4×3=12

(*a*) Role of algae in agriculture

(*b*) Evolutionary significance of Prochloron

(*c*) Role of bacteria in industry

(*d*) Role of virus in vaccine production

3. Give a detailed account of the range of thallus structure in algae with suitable diagrams. 8+4=12

Or

What is meant by 'alternation of generation'? Explain it with reference to the life history of Polysiphonia. How are the spores dispersed in this plant? 2+8+2=12

4. Describe the characteristics of Mycoplasma. How are they different from bacteria and viruses? Mention some of the diseases caused by PPLO (Pleuropneumonia-like organisms). 4+4+4=12

Or

Answer/Write explanatory note on the following : 6×2=12

(*a*) "Bacteria are both good and bad associates of human civilization." Justify the statement.

(*b*) Phases of bacterial growth curve

5. What are viruses? Are they living or non-living agents? Write about the methods of their transmission and the control measures of a typical plant viral disease. 1+3+4+4=12

Or

What are viroids and prions? How are they different from a typical virus? Draw and describe the structure of tobacco mosaic virus. 2+2+2+2+4=12

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1 SEM TDC CHMH (CBCS) C 2

2021

(Held in January/February, 2022)

CHEMISTRY

(Core)

Paper : C–2

(Physical Chemistry)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×3=3

(a) At 27 °C, the kinetic energy of two moles of N_2 is

(i) 7482·6 J

(ii) 3741·3 J

(iii) 2494·2 J

(iv) 0

(b) With the increase in temperature, the viscosity of a gas

(i) increases

(ii) decreases

(iii) at first increases then decreases

(iv) remains same

(c) Out of the four liquids given below, the one having lowest vapour pressure at 25 °C is

(i) carbon tetrachloride

(ii) benzene

(iii) chloroform

(iv) water

2. Answer any *four* from the following questions : 2×4=8

(a) Explain why we have to define the heat capacity of gases under constant pressure and constant volume condition.

(b) Write the SI unit of van der Waals' constants *a* and *b*. Mention the physical significance of *a*.

(c) At what temperature the root mean square speed of nitrogen at 27 °C would be tripled?

(d) Show that in face-centered cubic lattice 74% space is occupied by lattice points.

(e) Explain why an aqueous solution of Na_2CO_3 is basic in nature.

UNIT—I

3. Answer any *two* from the following questions : 7×2=14

(a) (i) Write kinetic gas equation. From this equation, derive Boyle's law. 1+2=3

(ii) Derive the relation between mean free path and coefficient of viscosity of a gas. Explain the effect of temperature on the viscosity of a gas. 3+1=4

(b) (i) What is equipartition of energy? In the light of it, calculate the total energy in joules associated with one mole of the following molecules at 27 °C : 1+1½+1½=4

(1) O_2

(2) H_2O

(ii) Calculate the volume of 5 moles of methane at 50 atmospheric pressure and 0 °C. At this temperature and pressure, the value of Z is 0·75. 2

(iii) An ideal gas can never be liquefied. Justify. 1

(c) (i) What is critical phenomenon? Derive the expression for the critical constants of a gas using van der Waals' equation of state. 1+3=4

(ii) For one mole of a gas, express van der Waals' equation in the virial form. 3

UNIT—II

4. Answer any *one* from the following questions : 5

(a) Define the term 'surface tension'. What is its SI unit? Describe the laboratory method for determining the surface tension of a liquid. 1+1+3=5

(b) (i) The time of flow of water through Ostwald viscometer is 1·48 minutes. For the same volume of a liquid of density 0·792 g/ml, it is 2·42 minutes. Find the viscosity of

the liquid relative to that of water and also absolute viscosity at 20 °C. Density and viscosity of water at 20 °C are 0·995 g/ml and 10·02 millipoise respectively. 2+2=4

(ii) Define coefficient of viscosity of a liquid. 1

UNIT—III

5. Answer any *two* from the following questions : 4½×2=9

(a) (i) Derive an expression showing the relation between the spacings of the lattice planes and the wavelength of the X-rays used to study the crystal system. 3

(ii) Draw (110) plane in a cubic crystal. 1½

(b) (i) Discuss Schottky defects and Frenkel defects of crystal giving examples. 3

(ii) Calculate the number of atoms present in a face-centered cubic unit cell. 1½

(c) (i) What are liquid crystals? Write two important properties each of nematic and smectic liquid crystals. 1+1+1=3

(ii) The structure of CsCl is different from that of NaCl though both have the similar formula. Give reason. 1½

UNIT—IV

6. Answer any *two* from the following questions : 7×2=14

(a) (i) What is salt hydrolysis? Prove that the aqueous solution of a salt formed by a strong acid and a weak base is acidic in nature. 1+3=4

(ii) Write any one difference between solubility and solubility product of calcium phosphate. 1+2=3

(b) (i) Deduce Henderson equation for acidic buffer and basic buffer solution. 2+2=4

(ii) Determine the pH of a solution obtained by mixing equal volumes of 0·015 *N* NH_4OH and 0·15 *N* NH_4NO_3 solutions. (pK_b of NH_4OH is 4·74) 3

(c) (i) What are acid-base indicators? Describe Ostwald theory of indicators taking the example of phenolphthalein. 1+3=4

(ii) Explain why methyl orange indicator is not used as indicator in the titration between a strong acid with a strong base. 1½

(iii) Calculate the pH of *N*/50 HCl solution. 1½

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Total No. of Printed Pages—7

1 SEM TDC CHMH (CBCS) C 1

2021

(Held in January/February, 2022)

CHEMISTRY

(Core)

Paper : C–1

(Inorganic Chemistry)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×6=6

(a) A golf ball has a mass of 40 g and a speed of 50 m/s. If the speed can be measured within accuracy of 2%, the uncertainty in the position is

(i) $1 \cdot 4 \times 10^{-31}$ m

(ii) $1 \cdot 4 \times 10^{-30}$ m

(iii) $1 \cdot 3 \times 10^{-30}$ m

(iv) $1 \cdot 3 \times 10^{-31}$ m

(b) Among the following, the third ionization energy is highest for

(i) Mg

(ii) Be

(iii) B

(iv) Al

(c) The pair of species with the same bond order is

(i) O_2^{2-} and B_2

(ii) O_2^+ and NO^+

(iii) NO and CO

(iv) N_2 and O_2

(d) The type of hybridization in NH_4^+ ion is

(i) sp^3

(ii) sp^2

(iii) sp

(iv) dsp^3

(e) The shape of SF_4 molecule is

(i) square planar

(ii) tetrahedral

(iii) *T*-shape

(iv) see-saw

(f) The oxidation number of chromium in potassium dichromate molecule is

(i) +6

(ii) +4

(iii) +3

(iv) +5

2. Answer the following questions : 2×9=18

(a) Write Schrödinger's wave equation and give the meanings of the symbols used there. 1+1=2

(b) What are normalised and orthogonal wave functions? 1+1=2

(c) Explain in the light of effective nuclear charge that Cl^- ion is larger in size than Cl atom. 2

(d) Zn and Cd have negative values of electron affinity. Explain why. 2

(e) Explain two factors on which the elctronegativity of an element depends. 2

(f) What will be the bond order and bond energy when you remove an electron from O_2 molecule? 2

(g) Using VSEPR theory, predict the structure of the following : 1+1=2

(i) ClF_3

(ii) XeF_2

(h) There is a decrease in bond angle from NH_3 (107°) to H_2O (104·5°) and an increase in bond angle from OF_2 (105°) to OCl_2 (111°). Explain why. 2

(i) Explain covalent character in ionic compounds with the help of Fajans' rules. 2

3. Answer any *two* of the following questions : 4×2=8

(a) (i) Draw the shapes of different *d*-orbitals.

(ii) Explain the contour boundary diagram. 2+2=4

(b) (i) An atom of an element contains 29 electrons and 35 neutrons. Deduce the number of protons and write the electronic configuration of the element. 1+1=2

(ii) Using $(n+l)$ rule, predict which of the following orbitals has highest energy :

5*p*, 5*d*, 5*f*, 6*s*, 6*p* 2

(c) (i) The velocity associated with a proton moving in a potential difference of 1000 V is $4{\cdot}37 \times 10^5$ ms^{-1}. If a hockey ball of mass 0·1 kg is moving with this velocity, calculate the wavelength associated with this velocity. 2

(ii) How many electrons in an atom may have the following quantum numbers? 1+1=2

1. $n = 4,\ m_s = -1/2$

2. $n = 3,\ l = 0$

4. Answer any *two* of the following questions : 3×2=6

(a) Mention two factors on which the electron affinity of an element depends. Explain why the electron affinity of F is lower than that of Cl. 1+2=3

(b) Mention two factors on which the ionization potential of an element depends. The 1st ionization potential of Be is higher than that of B, while the 2nd ionization potential of Be is lower than that of B. Explain. 1+2=3

(c) Calculate the effective nuclear charge for the last electron of sodium ion. How do effective nuclear charges vary while moving down a group? 1+2=3

5. Answer any *four* of the following questions : 3×4=12

(a) What is Born-Haber cycle? Show with example, how the lattice energy can be obtained. 1+2=3

(b) Draw the resonating structures of the following molecules and ions : 1+1+1=3

(i) SO_4^{2-}

(ii) $RCOO^-$

(iii) C_6H_6

(c) Discuss inter- and intra-molecular hydrogen bonding with suitable examples. 3

(d) Draw the molecular orbital energy-level diagram for N_2 molecule and calculate the bond order. 2+1=3

(e) Explain the following in the light of hydrogen bonding : 1+1+1=3

(i) Density of ice is lower than water

(ii) Glycerol is more viscous than glycol

(iii) Salicylic acid is insoluble in water

6. What is standard electrode potential? Explain two applications of standard electrode potential in inorganic reactions. 1+2=3

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Total No. of Printed Pages—3

1 SEM TDC ZOOH (CBCS) C 1

2021

(Held in January/February, 2022)

ZOOLOGY

(Core)

Paper : C–1

(Non-Chordates—I)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions.

1. Fill in the blanks : 1×5=5

(a) Amoebic dysentery is caused by ____.

(b) Canal system is found in ____.

(c) Interstitial cells in Hydra form ____.

(d) In jellyfish, the mouth is at the end of the ____.

(e) Excretion in flatworms is performed by ____.

2. Distinguish between any *two* of the following : 3×2=6

(a) Pinacoderm and Spicules

(b) Polyp and Medusa

(c) Platyhelminthes and Nemathelminthes

3. Write short notes on any *three* of the following : 3×3=9

(a) Canal system in sponges

(b) Locomotion in protists

(c) Coral reefs

(d) Ctenophora

4. Explain the sexual and asexual reproduction of protist. 6

5. Describe briefly the food trapping by sponge cells. 5

Or

Discuss the polymorphism in Cnidaria.

6. Explain the evolutionary significance of Ctenophora. 5

7. Write the general characters of Metazoa. 4

Or

Explain in brief the pathogenicity of *Taenia solium*.

8. Describe the life cycle of Plasmodium and its pathogenicity. 4+3=7

9. Describe in brief the life cycle and pathogenicity of *Ascaris lumbricoides*. 3+3=6

Or

Mention three characters and give the classification of Platyhelminthes up to class. 3+3=6

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1 SEM TDC ZOOH (CBCS) C 2

2021

(Held in January/February, 2022)

ZOOLOGY

(Core)

Paper : C–2

(Principle of Ecology)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Select the correct answer from the following : 1×5=5

(a) Which in the following is a proper food chain shown a producer, a herbivore and a carnivore?

(i) Grass—Insect—Elephant

(ii) Plants—Rabbit—Tiger

(iii) Fish—Insect—Whale

(iv) Tiger—Rabbit—Owl

(b) In which year, the concept of an ecosystem was first formally proposed by the English botanist Arthur Tansley?

(i) 1932

(ii) 1935

(iii) 1938

(iv) 1972

(c) Denitrifying bacteria change

(i) nitrite to nitrate

(ii) nitrate to nitrogen molecule

(iii) nitrate to nitrite

(iv) nitrogen to nitrate

(d) Biosphere refers to

(i) plants of the world

(ii) area occupied by living beings

(iii) special plants

(iv) plants of particular area

(e) The study of interrelationship between a species and its environment is called

(i) forest ecology

(ii) autecology

(iii) synecology

(iv) niche

2. (a) Write short notes on any *two* of the following : $4 \times 2 = 8$

(i) Survivorship curves

(ii) r and K strategies

(iii) Lotka-Volterra equation for competition and predation

(b) Write brief notes on any *two* of the following : $3 \times 2 = 6$

(i) Ecological pyramids

(ii) Ecological succession with hydrosere

(iii) Nitrogen cycle

3. Write a note on the importance of wildlife conservation. 5

Or

Explain any five abiotic factors of ecosystem.

4. Define ecosystem. Write about the different types of ecosystem and in detail about forest ecosystem. 1+2+4=7

Or

What is food chain? Describe Y-shaped food chain. 2+5=7

5. Answer the following questions :

(a) Define population growth. Describe the exponential and logistic growth curve, its equations and patterns. 2+10=12

(b) What is Gauss' principle in ecology? Describe it with laboratory and field examples. 2+4+4=10

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1 SEM TDC PHYH (CBCS) C 1

2021

(Held in January/February, 2022)

PHYSICS

(Core)

Paper : C–1

(Mathematical Physics—I)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) The partial derivative of $ye^{2x}+2xy^2$ is

(i) $2(ye^{2x}+xy^2)$

(ii) $2(ye^{2x}+y^2)$

(iii) $(ye^{2x}+2y^2)$

(iv) None of the above

(b) The degree and order of the differential equation

$$\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^2 + 2y = e^{3x}$$

are

(i) 2 and 2

(ii) 2 and 1

(iii) 1 and 2

(iv) None of the above

(c) If $\vec{A}$ is an irrotational vector, then

(i) $\vec{\nabla} \cdot \vec{A} = 1$

(ii) $\vec{\nabla} \times \vec{A} = 0$

(iii) $\vec{\nabla}\vec{A} = 0$

(iv) None of the above

(d) By Gauss divergence theorem, $\int_V \vec{\nabla} \cdot \vec{A} dV$ equals to

(i) $\int_S \vec{A} \cdot d\vec{S}$

(ii) $\oint_C \vec{A} \cdot d\vec{r}$

(iii) $\oint_C \vec{A} \cdot d\vec{S}$

(iv) None of the above

(e) A normal to the surface $\phi(x, y, z) = c$ is given by

(i) $\vec{\nabla} \cdot \phi$

(ii) $\vec{\nabla} \times \phi$

(iii) $\vec{\nabla}\phi$

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) Show that $\lim_{x \to 0} \sin\frac{1}{x}$ does not exist.

(b) For what values of a, $\vec{A}$ and $\vec{B}$ are perpendicular if $\vec{A} = a\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2a\hat{i} + a\hat{j} - 4\hat{k}$?

(c) What is a Wronskian? How is it used to find the linear dependence of two functions?

(d) Show that $\vec{B}$ is perpendicular to $\vec{A}$, if $|\vec{B}| \neq 0$ and $\vec{B} = \frac{d\vec{A}}{dt}$.

(e) Evaluate using the property of Dirac delta function :

$$\int_{-\infty}^{+\infty} x\delta(x-4)dx$$

3. Answer any *five* questions from the following : 4×5=20

(a) What do you mean by linearly dependent and linearly independent solutions of a homogeneous equation? If $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are two solutions of $y'' + 9y = 0$, then show that $y_1(x)$ and $y_2(x)$ are linearly independent solutions. 1+3=4

(b) If $z(x+y) = x^2 + y^2$, then show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$ 4

(c) Solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

Hence find the solution for

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$$ 3+1=4

(d) What is directional derivative? Find the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ at $(1, 1, -1)$ in the direction $2\hat{i} + \hat{j} - \hat{k}$. 1+3=4

(e) State Bayes' theorem of probability. 6 cards are drawn from a pack of 52 cards. What is the probability that 3 will be red and 3 black? 1+3=4

(f) State Green's theorem in a plane. Starting from Green's theorem, show that the area bounded by a closed curve is given by

$$\frac{1}{2}\oint_C (x\,dy - y\,dx)$$ 1+3=4

4. Answer any *three* questions from the following : 6×3=18

(a) What are complementary function and particular integral of a differential equation? Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2$$

if $y(0) = 0$ and $y'(0) = \frac{1}{2}$. 1+5=6

(b) Define line integral and surface integral. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along a curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. 2+4=6

(c) Show that $F = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Also find the work done in moving an object from (1, –2, 1) to (3, 1, 4). 2+2+2=6

(d) What are curvilinear coordinates? Describe the term 'scale factor' in curvilinear coordinates. Derive the expression for divergence of a vector in curvilinear coordinates. Hence write its expression in spherical polar coordinates. 1+2+3=6

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1 SEM TDC PHYH (CBCS) C 2

2021

(Held in January/February, 2022)

PHYSICS

(Core)

Paper : C–2

(Mechanics)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×5=5

(a) The angle of projection for a projectile whose equation of motion represented by $y = \frac{x}{\sqrt{3}} - \frac{g}{20}x^2$ is given by

(i) 59°

(ii) 60°

(iii) 30°

(iv) 65°

(b) In case of perfectly inelastic collision

(i) momentum is not conserved, KE is conserved

(ii) momentum is conserved, KE is not conserved

(iii) both momentum and KE are conserved

(iv) both momentum and KE are not conserved

(c) Mass of a body measured by a spring balance is

(i) inertial mass

(ii) gravitational mass

(iii) both gravitational and inertial mass

(iv) None of the above

(d) If the displacement of a particle executing SHM is represented by

$$y = 10\sin\left(10t - \frac{\pi}{6}\right)$$

then maximum velocity of the particle is

(i) 10

(ii) 100

(iii) 60

(iv) 600

(e) A photon is moving with a velocity c in frame S' and S' is moving with velocity c relative to frame S, then the velocity of the photon as measured by an observer in frame S is

(i) $2c$

(ii) c

(iii) c^2

(iv) None of the above

2. *(a)* Prove that the change in kinetic energy of a particle is equal to the work done on it. 2

(b) The position vector of a particle of mass m and angular velocity ω at any instant t is given by $\hat{r} = A\cos\omega t\,\hat{i} + A\sin\omega t\,\hat{j}$, where A is a constant. Prove that the force acting on it is conservative in nature. 2

Or

Show that the angular momentum of a body moving under the central force is a constant of motion.

(c) Two bodies of masses 10 gm and 5 gm have position vectors $(2\hat{i} + 3\hat{j} - \hat{k})$ and $(\hat{i} - \hat{j} + 3\hat{k})$ respectively. Determine the distance of the centre of mass from the origin. 2

Or

Show that for a simple harmonic oscillator the total mechanical energy is constant.

(d) Show that the moment of inertia of a body rotating about an axis with unit angular velocity is equal to twice the kinetic energy of rotation about that axis. 2

(e) What is Poisson's ratio? Explain why its practical value cannot be negative. 1+1=2

3. Obtain the expression for the final velocity of a rocket moving in a constant gravitational field. 4

4. *(a)* Obtain the expression for the moment of inertia of a hollow cylinder about an axis passing through its centre and perpendicular to its own axis. 4

Or

State the general theorems on moment of inertia. Assuming earth to be a sphere of uniform density ρ and radius R, determine the moment of inertia of the earth about its axis of rotation. 2+2=4

(b) Prove that the acceleration of a body of circular symmetry rolling down in an inclined plane without slipping is independent of mass of the body. 3

5. Write the main assumptions in deriving the Poiseuille's formula. Why this formula fails in the case of a tube of wide bore? 2+1=3

6. (a) Find an expression for the gravitational potential due to a solid sphere at the centre of the sphere. 4

Or

What is gravitational potential? Describe how the gravitational potential and field due to a spherical shell vary with distance graphically. 1+3=4

(b) What are geostationary and polar satellites? Which satellite can be used for remote sensing? 2+1=3

7. What is forced vibration? Obtain the steady-state function for such vibration. 1+4=5

8. Obtain an expression for Coriolis force for a particle moving with respect to a rotating frame. 4

9. (a) Describe the Michelson-Morley experiment and explain the physical significance of negative results. 4

Or

Deduce the formula for relativistic variation of mass with velocity. 4

(b) Show that the rest mass of a photon is zero. 2

(c) Find an expression for the relativistic length contraction. 2

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1 SEM TDC MTMH (CBCS) C 1

2021

(Held in January/February, 2022)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Write the value of $\frac{d}{dx}(\cosh x)$. 1

(b) Inverse hyperbolic sine is symmetric about a line. Write that line. 1

(c) Write the value of y_n, if $y = \cos(4x+3)$. 1

(d) Define point of inflection. 1

(e) Find $\frac{d}{dx}(\tanh\sqrt{1+x^2})$. 2

(f) Show that $\sinh x$ is an increasing function of x. 2

(g) Show that $y = x^2$ is concave up on $(-\infty, \infty)$. 2

(h) Show that $\operatorname{cosech}^{-1} x = \sinh^{-1}\frac{1}{x}$. 3

Or

Find the asymptotes of

$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0$

(i) Find y_n, if $y = \sin^3 x$. 3

Or

Find y_n, if $y = x^3 \sin x$.

(j) Evaluate (any *one*) : 4

(i) $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

(ii) $\lim_{x \to \frac{\pi}{2}} \frac{\tan 5x}{\tan x}$

2. (a) Find $\int \tan^5 x \, dx$. 3

Or

Evaluate $\int_0^1 x^2 (1-x)^{\frac{3}{2}} dx$.

(b) Obtain the reduction formula for

$\int \sin^n x \, dx$ 4

(c) Obtain the reduction formula for

$\int x^n e^{ax} \, dx$ 4

Or

Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

(d) Find the volume of the solid generated by revolving the region bounded by the curves and lines $y = x$, $y = -\frac{x}{2}$, $x = 2$ about the y-axis. 4

3. (a) Write the equation $x^2 + y^2 = 1$ in parametric form. 1

(b) A function $y = f(x)$ is defined on $[a, b]$. Write the domain of the function after given a natural parametrization

$x = t, \; y = f(t)$ 1

(c) Write the parametric formula for $\frac{d^2y}{dx^2}$. 1

(d) Write the equivalent Cartesian equation of the polar equation $r\cos\theta = 2$. 1

(e) Find the eccentricity of the ellipse $2x^2 + y^2 = 2$. 2

(f) Find the polar equation of $xy = 1$. 2

(g) Find the Cartesian equation from the parametric equation

$x = 4\cot t, \; y = 2\sin t, \; 0 \le t \le 2\pi$ 3 *(h)* Find a parametrization for the curve having the lower half of the parabola $x-1=y^2$. 4

Or

Find an equation for the line tangent to the curve $x=2\cos t$, $y=2\sin t$ at the point $t=\frac{\pi}{4}$.

4. *(a)* Define limit of a vector valued function. 1

(b) Let the position of a moving particle is given by

$$\vec{r}(t)=(\sec t)\hat{i}+(\tan t)\hat{j}+\frac{t^3}{3}\hat{k}$$

Find the acceleration at any time t. 2

(c) Evaluate the integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}[(\sin t)\hat{i}+(1+\cos t)\hat{j}+(\sec^2 t)\hat{k}]\,dt$$ 3

(d) Write the value of $[\vec{a}\ \vec{b}\ \vec{a}]$. 1

(e) Let $\vec{U}(t)$ and $\vec{V}(t)$ are differentiable vector function of t. Show that

$$\frac{d}{dt}(\vec{U}\cdot\vec{V})=\vec{U}'\cdot\vec{V}+\vec{U}\cdot\vec{V}'$$ 3

Or

Find the normal component of acceleration of a moving particle.

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1 SEM TDC MTMH (CBCS) C 2

2021

(Held in January/February, 2022)

MATHEMATICS

(Core)

Paper : C–2

(Algebra)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the complex number $\sqrt{2}(1+i)$ in the polar form. 1

(b) Find the equation whose roots are the nth power of the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$. 2

(c) Let $\text{cis}\theta = \cos\theta + i\sin\theta$. If $x = \text{cis}\,\alpha$, $y = \text{cis}\,\beta$, $z = \text{cis}\,\gamma$ and $x + y + z = xyz$, then show that

$$1 + \cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = 0$$ 3

Or

If α denotes any nth roots of unity, then show that $1+\alpha+\alpha^2+......+\alpha^{n-1}=0$.

(d) Using De Moivre's theorem, find the expansions of $\cos n\theta$ and $\sin n\theta$ where $n \in \mathbb{N}$ and hence deduce the expansions of $\cos\alpha$ and $\sin\alpha$ in powers of α. 4

2. *(a)* State whether true or false : 1

Union of two transitive relations is a transitive relation.

(b) Consider the functions $f:\mathbb{N}\to\mathbb{Z}$ defined by $f(n)=-2n$ and $g:\mathbb{N}\to\mathbb{R}$ defined by $g(n)=\frac{1}{n}$. Investigate the existence of $g\circ f$ justifying your assortion. 1

(c) Show that if $a \equiv b(\text{mod } m)$ and $c \equiv d(\text{mod } m)$, then

$$a+c \equiv b+d\ (\text{mod } m)$$ 2

(d) Define an injective mapping. Show that the mapping $f:\mathbb{Z}\to\mathbb{Z}$ defined by $f(x)=2x$ is injective. 2

(e) Let n be a non-zero fixed integer. For any integers a and b, define a relation $a \equiv b(\text{mod } n)$ if and only if n divides $a-b$. Show that this relation is an equivalence relation. 4

Or

Show that intersection of two equivalence relations on a set is again an equivalence relation.

(f) State and prove the well ordering property of the set of positive integers. 4

Or

Show by the principle of mathematical induction that

$$1^2+2^2+....+n^2=\frac{n}{6}(n+1)(2n+1)$$

(g) Let $f:A\to B$; $g:B\to C$; $h:C\to D$ be mappings. Show that

$$h\circ(g\circ f)=(h\circ g)\circ f$$ 3

(h) Let $\text{g.c.d.}(a, b)=1$. Show that

$$\text{g.c.d.}(a+b, a^2-ab+b^2)=1 \text{ or } 3$$ 4

(i) Let a and b be two integers. Suppose either $a\neq 0$ or $b\neq 0$. Show that there exists a greatest common divisor d of a, b such that $d=ax+by$ for some integers x and y which is uniquely determined by a and b. 4

3. (a) State whether true or false : 1

"Finding the parametric description of the solution set of a linear system is the same as solving the system."

(b) State which of the following statement/statements is/are false : 1

(i) The weights $c_1, c_2, \ldots, c_n$ in a linear combination $c_1v_1 + c_2v_2 + \ldots c_nv_n$ of vectors $v_1, v_2, \ldots, v_n$ can not all be zero.

(ii) Another notation of the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ is $[a, b]$.

(iii) An example of a linear combination of vectors v_1 and v_2 is $\frac{1}{2}v_1$.

(iv) None of the above are true.

(c) Given $A = \begin{bmatrix} -2 & -6 \\ 7 & 21 \\ -3 & 9 \end{bmatrix}$.

Find one non-trivial solution of $Ax = 0$. 2

(d) Show that the vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ are linearly dependent. 2

(e) Give the geometrical interpretation of span$\{u, v\}$ where

$$u = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \text{ and } v = \begin{bmatrix} 0 \\ 1 \\ 6 \end{bmatrix}$$

Indicate the subspace represented by the span. 2

(f) Define linear independence of vectors. Show that the columns of the matrix

$$A = \begin{bmatrix} 4 & 3 \\ -2 & -3 \\ 6 & 9 \end{bmatrix}$$ are linearly independent.

1+2=3

(g) Show that if an indexed set $S = \{v_1, \ldots, v_n\}$ with $n \geq 2$, is linearly dependent and $v_1 \neq 0$, then some v_j with $j > 1$ is a linear combination of the preceding vectors $v_1, \ldots, v_{j-1}$. 4

(h) Transform the augmented matrix represented by the linear system,

$$x_1 + 3x_2 + x_3 = 0$$
$$-4x_1 - 9x_2 + 2x_3 = 0$$
$$-3x_2 - 6x_3 = 0$$

(i) to Echelon form indicating the forward phase of row operations.

(ii) to reduced row Echelon form by indicating the backward phase of row operations.

Hence, indicate the basic variables and the free variables. 2+2+1=5

4. (a) Define a linear transformation. 1

(b) Show that $T(0)=0$ where $T:V\to W$ is a linear transformation. 1

(c) Investigate whether the following transformation is linear or not :

$T:\mathbb{R}^2\to\mathbb{R}^2$ defined by

$T(x_1, x_2)=(x_1+4, x_2)$ 2

(d) If A is an $n\times n$ invertible matrix, determine the column space of A and null space of A. 2

(e) Let $T:\mathbb{R}^n\to\mathbb{R}^m$ be linear. Show that T is one-to-one if and only if the equation $T(x)=0$ has trivial solution. 3

(f) By reducing the matrix

$$\begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{bmatrix}$$

to Echelon form, find the number of pivot columns and the rank. 3

Or

Find the characteristic equation of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

and the eigenvalues.

(g) Given a linear transformation $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(x)=Ax$ where

$$A=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Find $T(u)$, $T(v)$ and $T(u+v)$ where $u=\begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $v=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and interpret the effect of the transformation geometrically. 2+2=4

(h) Find a basis for the null space of the matrix

$$A=\begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix}$$ 4

Or

If $v_1, \ldots, v_p$ are eigenvectors corresponding to distinct eigenvalues $\lambda_1, \ldots, \lambda_p$ of an $n\times n$ matrix A, then show that the set $\{v_1, \ldots, v_p\}$ is linearly independent.

(i) Let A be an invertible matrix. Show that

(i) $(A^{-1})^{-1} = A$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$ 2+3=5

Or

Let $v_1, \ldots, v_p \in \mathbb{R}^n$. Show that the set of all linear combinations of $v_1, \ldots, v_p$ is a subspace of $\mathbb{R}^n$.

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