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**5 SEM TDC DSE MTH (CBCS) 1.1/1.2/1.3 (H)**

**2023**

( November )

# MATHEMATICS

( Discipline Specific Elective )

( For Honours )

Paper : DSE–1

*Full Marks* : 80

*Pass Marks* : 32

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

Paper : DSE–1.1

## ( Analytical Geometry )

**1.** Answer the following questions :

(a) Write the centre of the conic

$$\frac{(x-2)^2}{9}+\frac{(y+1)^2}{16}=1$$ 1

(b) Write the processes to sketch the hyperbola. 4

*(c)* Find the centre, foci, vertices and length of the major axis of the conic

$$4x^2 + y^2 + 8x - 2y + 1 = 0$$ 4

*(d)* Describe the graph of the curve

$$16x^2 + 9y^2 - 64x - 54y + 1 = 0$$

Also find its centre and foci. 6

*Or*

Describe the graph of the hyperbola

$$x^2 - y^2 - 4x + 8y - 21 = 0$$

and sketch its graph.

**2.** Answer the following questions :

*(a)* Write the condition of tangency of the line $y = mx + c$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ 1

*(b)* Write True or False : 1

A parabola is the set of all points in the plane that are equidistant from a fixed line and a fixed point not on the line.

*(c)* Define an ellipse. 1





*(d)* Find the equation of the ellipse, one of whose foci is $(-1, 1)$, eccentricity is $\frac{1}{2}$ and the corresponding directrix is $y = x + 3$. 5

*(e)* Find the equation of the parabola whose axis is parallel to the $y$-axis that has its vertex at $(5, -2)$ and passes through the point $(9, 5)$. Also sketch it. 7

*Or*

Find and sketch the curve of the hyperbola whose foci $(6, 4)$ and $(-4, -4)$ and eccentricity is 2.

**3.** Answer the following questions :

*(a)* Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents parabola. 1

*(b)* Determine a rotation angle $\theta$ that will eliminate the $xy$-term of the conic

$$2x^2 + \sqrt{3}xy + y^2 - 10 = 0$$ 2

*(c)* Consider the equation

$$x^2 - 4xy - 2y^2 - 6 = 0$$

Rotate the coordinate axes to remove the $xy$-term, then identify the type of conic represented by the equation and sketch its graph. 6

*(d)* Let an $x'y'$-coordinate system be obtained by rotating an $xy$-coordinate system through an angle $\theta = 45°$. Find an equation of the curve $3(x')^2 + (y')^2 = 6$ in $xy$-coordinate. 6

*Or*

Identify and sketch the curve

$$52x^2 - 72xy + 73y^2 + 40x + 30y - 75 = 0$$

**4.** Answer the following questions :

*(a)* Write the general equation of sphere. 1

*(b)* Write True or False : 1

The section of a sphere by a plane is a sphere.

*(c)* Write the standard equation of hyperbola of two sheets. 1

*(d)* Find the centre and radius of the sphere

$$x^2 + y^2 + z^2 - 8x + 4y - 6z + 4 = 0$$ 2

*(e)* A plane passes through a fixed point $(a, b, c)$ and meets the axes $A$, $B$, $C$. Show that the locus of the sphere $OABC$ is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$, where $O$ is the origin. 5





*(f)* Find the equation of the sphere through the origin and intersecting coordinate axes at distances $a$, $b$, $c$ from the origin. 5

*Or*

Obtain the equation of the sphere circumscribing the tetrahedron by the plane $x = y = z = 0$ and $2x + 3y + 4z - 12 = 0$.

**5.** Answer the following questions :

*(a)* Find the radius and the centre of the circle

$$x^2 + y^2 + z^2 + 12x - 13y - 16z + 111 = 0,$$
$$2x + 2y + z = 17$$ 5

*(b)* Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,$$
$$2x + 3y - 4z = 8$$

is a great circle. 5

*Or*

Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0,$$

also find their point of contact.

6. Answer the following questions :

(a) Find the condition that the plane $lx + my + nz = p$ may touch the sphere

$$x^2 + y^2 + z^2 + 2gx + 2fy + 2hx + d = 0$$ 5

(b) Classify and sketch the quadric surface (any *one*) : 5

(i) $\frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{16} = 1$

(ii) $z = \frac{x^2}{8} + \frac{y^2}{2}$



( Continued )



Paper : DSE–1.2

**( Portfolio Optimization )**

1. Answer any *five* of the following questions : 1×5=5

(a) What is risk averse?

(b) What is portfolio?

(c) What is the value of variance of risk-free investment?

(d) Define business risk.

(e) Define risk-free asset.

(f) What is mutual fund?

2. (a) If an investment that costs \$300 and is worth \$350 after being held for two years, find annual holding period return (annual HPR) and annual holding period yield (annual HPY). 4

(b) Define expected return of an investment. Calculate the expected rate of return of the following economic scenarios : 1+2=3

| *Economic Conditions* | *Probability* | *Rate of Return* |
|---|---|---|
| Strong economy | 0·25 | 0·20 |
| Weak economy | 0·25 | –0·20 |
| No major change in economy | 0·50 | 0·10 |



( Turn Over )

(c) Write the measures of risk in terms of variance and standard deviation of the estimated distribution of expected returns. Define the relative measure 'coefficient of variation' of risk. 2+2+1=5

(d) Describe different types of risk of an investment. 5

*Or*

Write three ways to change the relationship between risk and the required rate of return for an investment.

(e) Describe the relationship between risk and return. 4

(f) Describe the investment objectives for 25 years old investors and 65 years old investors. 4

**3.** (a) State one-fund theorem. 2

(b) What are the assumptions of the Markowitz portfolio theory? 5





(c) Write the formula for the expected return for a portfolio of investments. Calculate the expected return of portfolio of risky assets given by the table : 1+2=3

| *Weight* ($w_j$) *(Percent of Portfolio)* | *Expected Security Returns* ($R_i$) |
|---|---|
| 0·20 | 0·10 |
| 0·25 | 0·11 |
| 0·25 | 0·12 |
| 0·30 | 0·13 |

(d) What are the variance and standard deviation of returns for an individual investment? Calculate the variance for an individual risky asset given by the following table : 2+2+3=7

| *Possible Rate of Return* ($R_j$) | *Expected Security Return* [$E(R_j)$] | *Probabilities* ($P_i$) |
|---|---|---|
| 0·08 | 0·103 | 0·35 |
| 0·10 | 0·103 | 0·30 |
| 0·12 | 0·103 | 0·20 |
| 0·14 | 0·103 | 0·15 |

*Or*

Describe variance and standard variation of returns for a portfolio of investments. 7

*(e)* Define risk-free portfolio using standard deviation of a portfolio of investments. 2

*(f)* Write short notes on any *two* of the following : 3×2=6

*(i)* Optimal portfolio

*(ii)* Efficient frontier

*(iii)* Changes of slope of SML

**4.** Answer any *three* of the following questions : 5×3=15

*(a)* Write five assumptions of capital market theory.

*(b)* Discuss the relations between risk and diversification of portfolio.

*(c)* Derive the equation of the capital asset pricing model (CAPM).

*(d)* Determine the expected rate of return with CAPM for the following five stocks :

| *Stock* | *Beta* |
|---|---|
| *A* | 0·70 |
| *B* | 1·00 |
| *C* | 1·15 |
| *D* | 1·40 |
| *E* | –0·30 |

where, economy's $RER = 0{\cdot}05$, and expected return on the market portfolio $E(R_M) = 0{\cdot}09$.





**5.** What is security market line (SML)? Draw a rough diagram of SML showing low risk, average risk and high risk of an investment. 1+2=3

**6.** Suppose that during the most recent 10 years period, the average annual total rate of return including dividends on an aggregate market portfolio was 14 percent ($\bar{R}_M = 0{\cdot}14$) and the standard deviation of annual rate of return for the market portfolio over past 10 years was 20 percent ($\sigma_M = 0{\cdot}20$). Examine the risk-adjusted performance of the following portfolios using Sharpe measure :

| *Portfolio* | *Average Annual Rate of Return* | *Standard Deviation of Return* |
|---|---|---|
| *D* | 0·13 | 0·18 |
| *E* | 0·17 | 0·22 |
| *F* | 0·16 | 0·23 |

Also, plot their Sharpe measures with capital market line (CML). 7

*Or*

Describe Treynor portfolio performance measure with example.

Paper : DSE–1.3

**( Financial Mathematics )**

( For 2020 batch or later )

**1.** *(a)* Let the supply function of an item is given by $5q+3p=65$. Write the inverse supply function. 1

*(b)* Let $P$ be the selling price of an item. Write the revenue after introduction of excise tax $T$. 1

*(c)* Let $c$ be the capital after $n$ years, where $r$ is the rate of interest. Write the present value. 1

*(d)* Let the demand and supply functions are given by $4q+p=11$, $5q-2p=4$ respectively. Find the equilibrium set. 2

*(e)* Find the solution of the recurrence equation $y_t = 5y_{t-1}+6$. Given $y_0 = \frac{5}{3}$. 5

*Or*

Find the present value of an annuity of $H$ for $N$ years, at given fixed interest $r$.





**2.** *(a)* Write the economic interpretation of stable equilibrium. 2

*(b)* For the supply and demand set

$$S = \{(q,\ p) : q = bp - a\} \text{ and}$$
$$D = \{(q,\ p) : q = c - dp\}$$

Find the equilibrium price. 2

*(c)* The supply and demand sets are given by $\{(q,\ p) : 3q - p = -16\}$, $\{(q,\ p) : q + p = 28\}$. Find the price sequence. 4

*Or*

Determine whether the Cobweb model predicts stable or unstable equilibrium for the market with $q^S(p) = 2p-3$ and $q^D(p) = 18 - p$.

**3.** *(a)* Define marginal cost. 1

*(b)* Let a firm has cost function

$$C(q) = 700 + 21q - 3q^2 + q^3$$

Show that its marginal cost is always positive. 2

(c) Let supply and demand functions are given by $3q-2p=12$ and $2q+3p=48$. An excise tax $T$ per unit is imposed. Determine when the revenue is maximum. 5

*Or*

Find the maximum and minimum values of the function $f(x)=x^4-8x^3+16x^2-5$ in the interval $[1, 5]$.

**4.** (a) Write the mathematical representation of elasticity of demand. 1

(b) Define marginal revenue. 2

(c) Let for an efficient small firm, the cost function is $C(q)=q^3-10q^2+110q+180$ and maximum production capacity per day is 12 units. Determine (i) profit function and (ii) break-even point. 2+2=4

(d) Discuss elasticity of the demand function. 5

*Or*

Let the demand set for an item is

$$D=\{(q, p): q^2(2+p^3)=200\}$$

Determine the values of $p$ where the demand is elastic.





**5.** (a) Define portfolio of an investor. 2

(b) Describe an economy with many industries. 4

(c) Find the extreme value of the function

$$f(x, y)=3x^2+2xy+2y^2-160x-120y+8$$ 6

*Or*

Let the demand for two items are $x=2-2p^x+3p^y$ and $y=9+3p^x-p^y$

The cost function

$$C(x, y)=8+x+2x^2-xy+y^2$$

Find the profit function.

**6.** (a) Define Leontief matrix. 2

(b) Describe return matrix. 2

(c) Describe how to make money with matrices with an example. 6

*Or*

Find the equation for production schedule $\overline{x}=(x_1, x_2)$ in terms of external demand $\overline{d}=(d_1, d_2)$, where input-output model with two industries is given by

$$A=\begin{bmatrix} 0\cdot5 & 0\cdot2 \\ 0\cdot3 & 0\cdot4 \end{bmatrix}$$

7. (a) Define derivative asset. 1

(b) Write one form of hedging. 1

(c) Write one use of comparison principle. 1

(d) Write by which investments are described. 1

(e) Define cash flow stream. 2

8. (a) Define nominal interest rate. 1

(b) Define discount factor. 1

(c) Let $r$ be the rate of interest per year. Write the amount after one year, if $p$ is the principal amount. 1

(d) Write the continuous compounding formula for present value. 2

(e) Describe compounding at various intervals. 4

*Or*

Let (100, 200, 300, 400) be the cash flow stream and the rate of interest is 10% for each period. Find the future value at the end of cash flow.

(f) Find the internal rate of return of the cash flow (1, 0, −1, 1). 5





Paper : DSE–1.3

**( Financial Mathematics )**

( For 2019 batch only )

UNIT—I

1. Answer the following questions : 1×4=4

(a) Define nominal interest rate.

(b) Write one form of hedging.

(c) Mention the comparison principle.

(d) Define simple interest rate.

2. Answer the following questions : 2×4=8

(a) Define cash flow stream.

(b) Write the continuous compounding formula for present value.

(c) Write the nature of growth under compound interest.

(d) Write about the time value of money.

3. Answer any *four* of the following questions : 6×4=24

(a) Find the internal rate of return of the cash flow (1, 0, 1, −1).

(b) Describe compounding at various levels.

(c) Find the present value of a cash flow stream.

(d) Find the corresponding effective rate for 3% compounded monthly.

(e) Derive the mathematical formula to calculate annuity.

4. Describe mortgages. 4

UNIT—II

5. Answer the following questions : 1×4=4

(a) Write for what purpose net present value is used.

(b) Define money market.

(c) Write when security is termed as zero coupon bond.

(d) Write True or False :

Annuities are traded.





6. Answer the following questions : 2×4=8

(a) Write the formula to calculate annuity.

(b) Define amortization.

(c) Write about face value of a bond.

(d) Describe price-yield curve.

7. Answer any *two* of the following questions : 4×2=8

(a) Compute future value of cash flow stream (−2, 2, 1·5, 1), the periods are years and interest rate is 10%.

(b) Describe maturity.

(c) Describe duration.

8. Answer any *four* of the following questions : 5×4=20

(a) Describe municipal bonds and corporate bonds.

(b) Derive the mathematical formulation of Macaulay duration.

(c) Describe immunization.

(d) An 8% bond with 15 years to maturity has a yield of 9%. Determine the price of this bond.

(e) Describe the derivation of variance of the rate of return of the portfolio.

(f) Describe two-fund theorem.

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**5 SEM TDC DSE MTH (CBCS) 2.1/2.2/2.3/2.4 (H)**

**2 0 2 3**

( November )

MATHEMATICS

( Discipline Specific Elective )

( For Honours )

Paper : DSE–2.1/2.2/2.3/2.4

*The figures in the margin indicate full marks for the questions*

Paper : DSE–2.1

**( Mathematical Modelling )**

*Full Marks* : 60
*Pass Marks* : 24

*Time* : 3 hours

1. (a) What do you mean by an ordinary point of the following differential equation?

$$a_0(x)\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)y=0$$ 1

(b) Define Bessel's equation of order zero. 1

2. *(a)* Show that $x = 0$ is a regular singular point of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$ 3

*(b)* Find the power series solution near $x = 0$ of the differential equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2+2)y = 0$$

in powers of $x$. 6

*Or*

Solve the following Bessel's equation :

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$$

3. *(a)* If $L\{F(t)\} = f(s)$, then prove that

$$L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$$ 2

*(b)* Prove that

$$L\{-a\sin at\} = -\frac{a^2}{s^2+a^2}$$ 3





*(c)* Evaluate the following using convolution theorem (any *one*) : 4

(i) $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$

(ii) $L^{-1}\left\{\frac{1}{(s-2)(s^2+1)}\right\}$

*(d)* Solve the initial value problem using Laplace transform, $y'' + y = t\cos t$ with $y(0) = 0$, $y'(0) = 0$. 5

4. *(a)* Write two characteristics of Monte Carlo simulation technique. 2

*(b)* Write the algorithm that gives the sequence of calculations needed for a general computer simulation of Monte Carlo technique for finding the area under a curve. 3

5. *(a)* Describe the middle-square method for generating random numbers. Write two disadvantages of middle square method. 3+2=5

*(b)* Use linear congruence method to generate a sequence of 10 random numbers with $x_0 = 27$, $a = 17$, $b = 43$ and $m = 100$ by the rule

$$x_{n+1} = (ax_n + b) \bmod (m)$$ 5

**6.** Write a short note on any *one* of the following : 5

*(a)* Morning rush hour queuing model

*(b)* Harbor model with example

**7.** Answer any *one* of the following : 6

*(a)* A firm makes two products $X$ and $Y$, and has a total production capacity of 9 tonnes per day. Both $X$ and $Y$ require the same production capacity. The firm has a permanent contract to supply at least 2 tonnes of $X$ and at least 3 tonnes of $Y$ per day to another company. Each tonne of $X$ requires 20 machine hours of production time and each $Y$ requires 50 machine hours of production time. The daily maximum possible number of machine hours is 360. All of the firm's output can be sold. The profit made is ₹ 80 per tonne of $X$ and ₹ 120 per tonne of $Y$. Solve the problem by using graphical method to determine the production schedule that yields the maximum profit.





*(b)* Using simplex method to solve the following linear programming model :

Maximize $Z = 4x_1 + 3x_2$

subject to

$$2x_1 + x_2 \le 1000$$
$$x_1 + x_2 \le 800$$
$$x_1 \le 400, \; x_2 \le 700$$

and $x_1, \; x_2 \ge 0$

**8.** A company wants to produce three products—$A$, $B$ and $C$. The per unit profit on these products is ₹ 4, ₹ 6 and ₹ 2 respectively. These products require two types of resources—manpower and raw material. The LP model formulated for determining the optimal product is as follows :

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

subject to the constraints

*(i)* $x_1 + x_2 + x_3 \le 3$ (Manpower constraint)

*(ii)* $x_1 + 4x_2 + 7x_3 \le 9$ (Raw material constraint)

where $x_1$, $x_2$, $x_3$ are the numbers of units of products $A$, $B$, $C$ respectively to be produced, and $x_1, x_2, x_3 \ge 0$.

*(a)* Find the optimal product mix and the corresponding profit of the company. 4

*(b)* Find the range of the profit contribution of product *A* in the objective function such that current optimal product mix remains unchanged. 5

Paper : DSE–2.2

**( Mechanics )**

*Full Marks* : 80

*Pass Marks* : 32

*Time* : 3 hours

**1.** *(a)* Determine the moment about the origin *O* of the force $\vec{F} = -5N\hat{i} - 2N\hat{j} + 3N\hat{k}$ which acts at a point *A*. The position vectors of *A* are *(i)* $\vec{r} = 4m\hat{i} - 2m\hat{j} - 1m\hat{k}$ and *(ii)* $\vec{r} = -8m\hat{i} + 3m\hat{j} + 4m\hat{k}$. 2+2=4

*(b)* Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting along $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, where *O* is the circumcentre of the triangle *ABC*, are in equilibrium. Show that

$$\frac{P}{a^2(b^2+c^2-a^2)} = \frac{Q}{b^2(c^2+a^2-b^2)} = \frac{R}{c^2(a^2+b^2-c^2)}$$

6





*Or*

Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting along $\overrightarrow{IA}$, $\overrightarrow{IB}$, $\overrightarrow{IC}$, where *I* is the incentre of the triangle *ABC*, are in equilibrium. Show that

$$P:Q:R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$$

*(c)* Show that two couples in the same plane whose moments are equal and of the same sign are equivalent to one another. 6

*(d)* An electric light fixture weighing 15 N hangs from a point *C*, by two strings *AC* and *BC*. *AC* is inclined at 60° to the horizontal and *BC* at 45° to the vertical. Draw the free body diagram and determine the forces in the strings *AC* and *BC*. 4

**2.** *(a)* Write down the Coulomb's laws of friction. 2

*(b)* An automobile is on a roadway inclined at an angle θ with the horizontals. If the coefficient of static and dynamic frictions between the tyres and road are 0·6 and 0·5 respectively, find the maximum inclination '$\theta_{max}$' at which car can climb at uniform speed. It has a rear-wheel drive and a total loaded weight of 3600 kg. 6

(c) Find $I_{xx}$, $I_{yy}$ and $I_{xy}$ for the area bounded by $y = e^x$ and $y = -e^x$. 2+2+2=6

*Or*

Find the centroid of the region bounded by $y^2 = 2x$, the line $\frac{x}{10} + \frac{y}{7} = 1$ and $y$-axis. 6

(d) State and prove the theorem of Pappus-Guldinus. 2+4=6

(e) Establish the relation between second moments and product of inertia. 5

3. (a) What do you mean by conservative force field? Show that in a conservative force field, $\vec{F} = -\nabla V$, where the symbols have their usual meanings. 2+3=5

(b) Show that the kinetic energy of a system for some reference is equal to the sum of kinetic energy of the total mass moving relative to that reference with the velocity of the mass centre and kinetic energy of the motion of the particles relative to the mass centre. 7





*Or*

Show that the moment of the resultant force on a particle about a point, fixed in an inertial reference, equal to the time rate of change of moment of the linear momentum of the particle relative to the inertial reference frame.

(c) Derive the moment of momentum equation for a system of particle. 6

(d) Establish the relationship between time derivatives of a vector for different references moving arbitrarily relative to each other. 6

(e) Show that the kinetic energy of a system of particles is equal to the sum of the kinetic energy of the mass centre and the kinetic energy of the system in its motion relative to moving frame of reference. 5

(f) For a given conservative force field

$$\vec{F} = (5z\sin x + y)\hat{i} + (4yz + x)\hat{j} + (2y^2 - 5\cos x)\hat{k}$$

find the force potential. What is the work done on a particle starting at the origin and moving in a circular path of radius 2 to form a semicircle along the positive $x$-axis? 6

*Or*

A solid cylinder of mass 20 kg rotates about its own axis with angular velocity of 100 rad/s, the radius of the cylinder is 0·25 m. Calculate the kinetic energy associated with the rotation of the cylinder.

Paper : DSE–2.3

**( Number Theory )**

*Full Marks* : 80
*Pass Marks* : 32

*Time* : 3 hours

1. *(a)* Write the Goldbach conjecture. 1

   *(b)* Can the Diophantine equation $14x+35y=93$ be solved? Give reasons to your answer. 2

   *(c)* Write the value of $\pi(30)$, where $\pi(x)$ denotes the prime counting function. 1

   *(d)* If $a \equiv b \pmod{m}$ and $x \equiv y \pmod{m}$, then prove that $ax \equiv by \pmod{m}$. 2

   *(e)* Find the remainder when 15! is divided by 17. 2





2. Answer any *three* of the following : 4×3=12

   *(a)* Find the general solution of the equation $5x+3y=52$.

   *(b)* Use Fermat's theorem to verify that 17 divides $11^{104}+1$.

   *(c)* Solve the simultaneous congruence :
   $$x \equiv 5 \pmod{7}$$
   $$x \equiv 7 \pmod{11}$$
   $$x \equiv 3 \pmod{13}$$

   *(d)* If $p$ be a prime number, then show that
   $$(p-1)! \equiv (p-1) \pmod{(1+2+3+\cdots+p-1)}$$

   *(e)* Prove that if $p$ is a prime, then $a^p \equiv a \pmod{p}$ for any integer $a$.

3. *(a)* Write the value of $\sigma(p)$, where $p$ is prime. 1

   *(b)* Prove that for each positive integer $n$, $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3)=0$. 2

4. Answer any *four* of the following : 3×4=12

   *(a)* Prove that for any integer $n>1$
   $$n^{\frac{\tau(n)}{2}} = \prod_{d|n} d$$

(b) If $F$ is a multiplicative function and

$$F(n) = \sum_{d|n} f(d)$$

then prove that $f$ is also multiplicative.

(c) Prove that the function $\sigma$ is multiplicative.

(d) Find the highest power of 7 that divides 2000!

(e) For $n > 2$, prove that $\phi(n)$ is even integer.

5. (a) Use Euler's theorem to establish any *one* of the following : 5

(i) For any integer $n \geq 0$, 51 divides $10^{32n+9} - 7$

(ii) For any integer $a$, $a^{37} \equiv a \pmod{1729}$, given that $1729 = 7 \times 13 \times 19$

(b) For any positive integer $n$, prove that

$$\phi(n) = n \sum_{d|n} \frac{\mu(d)}{d}$$

Also verify it for $n = 12$. 5

(c) Define Dirichlet product of arithmetic functions. Also prove that $(f * g) * h = f * (g * h)$, where $f$, $g$ and $h$ are arithmetic functions and '$*$' denotes their Dirichlet product. 5





*Or*

Prove that if $f$ and $g$ are both multiplicative functions, then their Dirichlet product $f * g$ is also multiplicative.

6. (a) Find the order of 5 modulo 12 and hence determine the order of $5^6$ modulo 12. 2

(b) Prove that if $a$ has order $2k$ modulo an odd prime $p$, then $a^k \equiv -1 \pmod{p}$. 3

7. Answer any *five* of the following : 5×5=25

(a) If $a$ is a primitive root of $m$, then show that $a^k$ is also a primitive root of $m$ if and only if $(k, \phi(m)) = 1$.

(b) Show that the primitive root of 13 are given by $S = \{2^n, 1 \leq n < \phi(m)\}$, $(n, \phi(m)) = 1$ when 2 is a primitive root of 13. Also find the exact number of primitive roots of 13.

(c) If $\gcd(m, n) = 1$, where $m > 2$ and $n > 2$, then prove that the integer $mn$ has no primitive roots.

(d) Write the Euler's criterion for quadratic residue of an odd prime. Find which of the integers $1, 2, 3, \cdots, 12$ are quadratic residues of 13 and which are non residues of 13.

*(e)* State quadratic reciprocity law. Also find the value of $\left(\frac{29}{53}\right)$.

*(f)* Solve the following quadratic congruence :

$$x^2 + 7x + 10 \equiv 0 \pmod{11}$$

*(g)* Define Legendre symbol $\left(\frac{a}{p}\right)$, where $p$ is an odd prime and $\gcd(a, p) = 1$. Prove that $a \equiv b \pmod{p} \Rightarrow \left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$. Hence show that $\left(\frac{3}{11}\right) = \left(\frac{14}{11}\right)$.

*(h)* Show that if $p$ is an odd prime, then

$$\sum_{a=1}^{p-1} \left(\frac{a}{p}\right) = 0$$

*(i)* Show that 7 and 18 are the only incongruent solution of $x^2 \equiv -1 \pmod{5^2}$.

*(j)* The message IWGA IU ZWU has been encoded with a Caesar cipher. Decipher it, using exhaustive cryptoanalysis.





Paper : DSE–2.4

**( Biomathematics )**

*Full Marks* : 80
*Pass Marks* : 32

*Time* : 3 hours

UNIT—I

1. Answer any *two* of the following questions : 7½×2=15

*(a)* A population is originally 100 individuals, but because of the combined effects of births and deaths, it triples each hour.

*(i)* Make a table of population size for $t = 0$ to 5, where $t$ is measured in hours.

*(ii)* Give two equations modelling the population growth by first expressing $P_{t+1}$ in terms of $P_t$ and then expressing $\Delta P$ in terms of $P_t$.

*(iii)* What can you say about the birthrate and death rate for this population?

*(b)* In the early stages of the development of a frog embryo, cell division occurs at a fairly regular rate. Suppose you

observe that all cells divide, and hence the number of cells double, roughly every half-hour.

(i) Write down an equation modelling this situation. You should specify how much real-world time is required by an increment of 1 in $t$ and what the initial number of cells is.

(ii) Produce a table and graph of the number of cells as a function of $t$.

(c) Obtain a simple prey-predator model explaining in detail the assumptions taken. Also find the equilibrium positions.

UNIT—II

**2.** Answer any *two* of the following questions : 7½×2=15

(a) Consider the SI epidemic model. If the contact rate is 0·001 and the number of susceptible is 2000 initially, determine—

(i) the number of susceptible left after 3 weeks;

(ii) the density of susceptible when the rate of appearance of new cases is a maximum;





(iii) the time (in weeks) at which the rate of appearance of new cases is a maximum;

(iv) the maximum rate of appearance of new cases.

(b) In an SIS model, if the infection is spread only by a constant number of carriers, then show that

$$I(t) = \left(I_0 - \frac{\alpha CN}{\alpha C+\beta}\right)e^{[-(\alpha C+\beta)t]} + \frac{\alpha CN}{\alpha C+\beta}$$

where $I$ and $C$ are the number of infectives and carriers; $N$ total population; $\alpha$ and $\beta$ are contact rate and susceptible rate respectively; $I_0$ is the infectives at $t = 0$.

(c) Let $x$ and $y$ respectively denote the proportion of susceptibles and carriers in a population. Suppose the carriers are identified and removed from the population at a rate $\beta$, so that $\frac{dy}{dt} = \beta y$.

Suppose also that the disease spreads at a rate proportional to the product of $x$ and $y$, thus

$$\frac{dx}{dt} = -\alpha xy$$

*(i)* Determine the proportions of carriers at any time $t$, where $y(0) = y_0$.

*(ii)* Use *(i)* to find the susceptibles at time $t$, where $x(0) = x_0$.

*(iii)* Find the proportion of population that escapes the epidemic.

UNIT—III

**3.** Answer any *two* of the following questions : 7½×2=15

*(a)* Consider the competition model for two species with populations $N_1$ and $N_2$ :

$$\frac{dN_1}{dt} = r_1 N_1\left(1 - \frac{N_1}{K_1} - b_{12}\frac{N_2}{K_1}\right)$$

$$\frac{dN_2}{dt} = r_2 N_2\left(1 - b_{21}\frac{N_1}{K_2}\right)$$

where only one species $N_1$, has limited carrying capacity. Investigate their stability and sketch the phase plane trajectories. [Here, $K_1$ and $K_2$ are carrying capacities; $r_1$ and $r_2$ are linear birthrates of the populations $N_1$ and $N_2$ respectively. $b_{12}$ and $b_{21}$ measure the competitive effect of $N_2$ on $N_1$ and $N_1$ on $N_2$ respectively.] 4+3½=7½





*(b)* What is Routh-Hurwitz criteria? Explain with reference to multiple species communities. 2+5½=7½

*(c)* Discuss bifurcation and limit cycle with respect to any biological model.

UNIT—IV

**4.** Answer any *two* of the following questions : 7½×2=15

*(a)* Write a short note on any *one* of the following :

*(i)* One species model with diffusion

*(ii)* Two species model with diffusion

*(b)* For a blood vessel of constant radius $R$, length $L$ and driving force $P = p_1 - p_2$, show that the average velocity of the flow is equal to half of the maximum velocity and the resistance is proportional to $\frac{L}{R^4}$.

*(c)* Consider the arterial blood viscosity $\mu = 0{\cdot}027$ poise. If the length of the artery is 2 cm, and radius $8 \times 10^{-3}$ cm and $P = p_1 - p_2 = 4 \times 10^3$ dynes/cm$^2$, then find—

(i) $q_z(r)$ and the maximum peak velocity of blood;

(ii) the shear stress at the wall.

Here $q_z$ denotes velocity along $z$-axis, $p_1$ and $p_2$ denote pressure at two ends of the artery.

UNIT—V

**5.** Answer any *two* of the following questions : 10×2=20

(a) Let D and d, and W and w respectively denote allele for tall and dwarf, and round and wrinkled seeds of peas. Find the outcome of the product DdWw × ddWw using Puneet square or using probability. Also find the probability that the progeny of DdWw × ddWw is dwarf with round seeds. 6+4=10

(b) Explain, in detail the Hardy-Weinberg equilibrium, mentioning the assumptions considered for the equilibrium.

(c) Compare and contrast stage structure model with age structure model.

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Total No. of Printed Pages—4

## 5 SEM TDC MTMH (CBCS) C 12

**2023**

( November )

# MATHEMATICS

( Core )

Paper : C–12

**( Group Theory—II )**

*Full Marks* : 80

*Pass Marks* : 32

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

1. (a) State True or False : 1

Every cyclic group is abelian.

(b) Define characteristic subgroup. 2

(c) If $\phi$ be an automorphism of a group $G$, then show that $H = \{x \in G \mid \phi(x) = x\}$ is a subgroup of $G$. 3

(d) Show that if $O(\text{Aut } G) > 1$, then $O(G) > 2$. 3

(e) Let $G$ be a group. Show that the mapping $\phi : G \to G$ such that $\phi(x) = x^{-1}$ $\forall x \in G$ is an automorphism if and only if $G$ is abelian. 4

*(f)* Let $T$ be an automorphism of $G$. Show that $O(Ta) = O(a)$ for $a \in G$. Deduce that $O(baa^{-1}) = O(a)$ for all $a, b \in G$. 5

**2.** Answer any *two* of the following : 6×2=12

*(a)* Determine Aut(G), where $G$ is Klein's 4-group.

*(b)* Prove that the characteristic subgroup of $G$ must be a normal subgroup of $G$. The converse need not be true.

*(c)* Let $I(G)$ be the set of all inner automorphisms of a group $G$. Then prove that

$$I(G) \approx \frac{G}{Z(G)}$$

**3.** *(a)* Express $U(165)$ as an external direct product of cyclic group of the form $Z_n$. 2

*(b)* Find the number of cyclic subgroups of order 10 in $Z_{10} \oplus Z_{25}$. 3

*(c)* If $m$ and $n$ are relatively prime, then prove that $U(mn) \approx U(m) \oplus U(n)$. 5

*Or*

Find the external direct product of the following two cyclic groups :

$G_1 = \{a, a^2 = e_1\}$ ; $G_2 = \{b, b^2, b^3 = e_2\}$





*(d)* Prove that a group $G$ is internal direct product of its subgroups $H$ and $K$ if and only if *(i)* $H$ and $K$ are normal subgroups of $G$ and *(ii)* $H \cap K = \{e\}$. 5

*(e)* Let $A$ and $B$ be cyclic groups of orders $m$ and $n$ respectively. Prove that $A \times B$ is cyclic if and only if $m$ and $n$ are relatively prime. 5

*Or*

Show that a group of order 4 is either cyclic or an internal direct product of two cyclic subgroups each of order 2.

**4.** *(a)* Write the class equation for a finite group $G$. 1

*(b)* If $a$ be an element of a group $G$, then show that $G$ is abelian, if and only if $Cl(a) = \{a\} \quad \forall\, a \in G$. 3

*(c)* Let $G$ be a finite group and $Z(G)$ be the centre of $G$. Then prove that

$$O(G) = O(Z(G)) + \sum_{a \notin Z(G)} \frac{O(G)}{O(N(a))}$$ 3

*(d)* If $G$ is a finite group, then prove that

$$O(G) = \sum \frac{O(G)}{O(N(a))}$$

where the sum is taken over one element of each conjugate class. 3

**5 SEM TDC MTMH (CBCS) C 11**

**2023**

( November )

MATHEMATICS

( Core )

Paper : C–11

**( Multivariate Calculus )**

*Full Marks* : 80
*Pass Marks* : 32

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

1. (a) State the domain of the function

$$w = xy\log z$$ 1

(b) Find the level curve of

$$f(x, y) = \int_y^x \frac{dt}{\sqrt{1+t^2}}$$

passing through the point $(\sqrt{2}, -\sqrt{2})$. 1

*(c)* Show that limit of the following functions does not exist (any *one*) : 3

*(i)* $\lim_{(x, y)\to(0, 0)} \frac{x}{\sqrt{x^2+y^2}}$

*(ii)* $\lim_{(x, y)\to(0, 0)} \frac{xy}{|xy|}$

*(d)* At what point/points $(x, y)$, is the function $f(x, y) = \log(x^2+y^2)$ continuous? Justify your answer. 3

*(e)* Show that $w_{xy} = w_{yx}$ where

*(i)* $w = \log(2x+3y)$

*(ii)* $w = x\sin y + y\sin x + xy$ 2+2=4

*Or*

Find the linearization of

$$f(x, y) = x^2 - xy + \frac{y^2}{2} + 3$$

at $(2, 3)$. 4

*(f)* Find $\frac{dw}{dt}$ at $t=1$ where $w = z - \sin xy$ and $x=t,\ y=\log t,\ z=e^{t-1}$. 4

*Or*

Find the derivative of $f(x, y, z) = xy + yz + zx$ at the point $(1, -1, 2)$ in the direction of $\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$.





*(g)* Find the tangent plane and normal to the surface $x^2+y^2-z^2=18$ at $(3, 5, -4)$. 4

*Or*

Prove that if $f(x, y)$ has a local extremum value at a point $(a, b)$ of its domain, and if the first partial derivatives exist there, then $f_x(a, b)=0$ and $f_y(a, b)=0$.

*(h)* Find the local extrema or saddle point as applicable of the function

$$f(x, y) = x^3 - y^3 - 2xy + 6$$ 5

*(i)* Use Lagrange's multipliers to maximize

$$f(x, y) = x^2 + 2y - z^2$$

subject to the constraints $2x - y = 0$ and $y + z = 0$. 5

*Or*

Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1, 1, 1)$.

**2.** *(a)* Sketch the region of integration of

$$\iint_R f(x, y)\,dA$$

on the plain paper, where the region $R$ is bounded by the line $-x+y=1$ and the curve $x^2+y^2=1$. 1

*(b)* Define cylindrical coordinates. 1

*(c)* State Fubini's theorem for a region $R$. 2

*Or*

Determine the limits of the double integral

$$\iint_R f dA$$

over the region $R$ bounded by the line $x+y=1$ and the curve $x^2+y^2=1$ while integrating at first with respect to $x$ and secondly, with respect to $y$.

*(d)* Evaluate $\int_0^1 \int_y^{\sqrt{y}} dxdy$ and then change the order of integration by drawing diagram. 1+3=4

*Or*

Evaluate $\iint_R (y-2x^2)\,dA$, where $R$ is the region inside the square $|x|+|y|=1$. 4

*(e)* Change the following into an equivalent polar integral and evaluate

$$\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$$ 4

*(f)* Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane

$$x+\frac{y}{2}+\frac{z}{3}=1$$ 4





*(g)* Evaluate : 4

$$\int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^{2} 3\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

**3.** *(a)* Define flux across a plane curve. 1

*(b)* State the fundamental theorem on line integrals. 2

*(c)* Use transformations $u=x-y$ and $v=2x+y$ to evaluate the integral

$$\iint_R (2x^2-xy-y^2)\,dx\,dy$$

where $R$ is the region bounded by the lines $y=-2x+4$, $y=-2x+7$, $y=x-2$ and $y=x+1$. 4

*Or*

Find the Jacobian

$$\frac{\partial(x, y, z)}{\partial(u, v, w)}$$

of the following transformations :

*(i)* $x=u\cos v;\ y=u\sin v;\ z=w$

*(ii)* $x=2u-1;\ y=3v-4;\ z=\frac{1}{2}(w-4)$

*(d)* Integrate

$$\int_C f(x, y)\,dS$$

where $f(x, y)=x+y$ and $C$ is the circle $x^2+y^2=4$ in the first quadrant from $(2, 0)$ to $(0, 2)$. 4

*(e)* If $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$, where $M$, $N$, $P$ are functions of $x$, $y$, $z$; be a field whose component functions have continuous first-order partial derivatives, then prove that $\vec{F}$ is conservative if and only if $P_y = N_z$, $M_z = P_x$ and $N_x = M_y$. 4

**4.** *(a)* Find the flux density of $\vec{F} = xz\hat{i} - xy\hat{j} - z\hat{k}$. 1

*(b)* Define surface integral. 2

*(c)* Integrate $G(x, y, z) = x^2$ over the sphere $x^2 + y^2 + z^2 = 1$. 3

*(d)* Let $C$ be a smooth closed and simple curve in the $xy$-plane with the property that the lines parallel to the axes cut it in no more than two points. Let $R$ be the region enclosed by $C$ and assume that the functions $M(x, y)$ and $N(x, y)$ and their first-order partial derivatives are continuous at every point of some open regions containing $C$ and $R$. Then show that

$$\oint_C (Mdx + Ndy) = \iint_R (N_x - M_y)\, dx\, dy$$ 4



*( Continued )*



*(e)* Prove that the flux of a vector field $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$, where $M$, $N$, $P$ are functions of $x$, $y$, $z$ across a closed piecewise smooth oriented surface $S$, in the direction of its outward unit normal field $\hat{n}$, is equal to

$$\iiint_D \nabla \cdot \vec{F} dV$$

where $D$ is the convex region without holes or bubbles. 5

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**5 SEM TDC DSE ZOO (CBCS) 3 (H)**

**2023**

( November )

ZOOLOGY

( Discipline Specific Elective )

( For Honours )

Paper : DSE–3

**( Endocrinology )**

*Full Marks* : 53
*Pass Marks* : 21

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Fill in the blanks : 1×5=5

*(a)* The posterior pituitary gland is composed of mainly _____ cells.

*(b)* _____ protein helps to transport iodide out of the thyroid cells into the follicle.

*(c)* The adrenal cortex secretes a group of hormones called _____.

(d) Single hormone secreted by pineal gland is _____.

(e) _____ cells secrete the testosterone hormone.

**2.** Write briefly on any *four* of the following : 4×4=16

(a) Hypothalamic nuclei and their functions

(b) Thyroid gland disorders

(c) Somatotropic hormone

(d) Neurosecretions

(e) Cushing's syndrome

**3.** Why are hormones called as chemical messenger? Briefly explain the chemical nature of hormone. 4+4=8

*Or*

Why is pineal gland called 'third eye'? How does the pineal gland regulate biological rhythm and reproduction? 2+(3+3)=8

**4.** Give an account of histology of pituitary gland and mention the functions of hormones secreted by neurohypophysis. 4+4=8

*Or*

Describe the histological structure of pancreas with suitable diagram and write the roles of pancreatic hormones in physiological activities. 3+5=8





**5.** What do you mean by gonadal hormone? Write the chemical natures and functions of estrogen and progesterone. 2+6=8

*Or*

Describe the histological structure of testis with suitable diagram and write the endocrine functions. 5+3=8

**6.** What do you mean by signal transducer? Explain the role of hormone in homeostasis. 2+6=8

*Or*

What are the common causes of endocrine gland disorders? Explain the disorders of adrenal gland and pituitary gland. 3+5=8

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Total No. of Printed Pages—3

**5 SEM TDC DSE ZOO (CBCS) 4 (H)**

**2023**

( November )

ZOOLOGY

( Discipline Specific Elective )

( For Honours )

Paper : DSE–4

**( Biology of Insecta )**

*Full Marks* : 53
*Pass Marks* : 21

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Fill in the blanks : 1×5=5

(a) The visual unit of a compound eye of an insect is ____.

(b) ____ legs are modified for leaping and jumping.

(c) Butterfly belongs to the order ____.

(d) _____ are the main osmoregulatory and excretory organs in insects.

(e) The body cavity of insects is called _____.

2. Write short notes on any *two* of the following : 3×2=6

   (a) General characters of class Insecta

   (b) Types of antennae

   (c) Allelochemicals

3. Answer any *three* of the following : 4×3=12

   (a) Male reproductive system of insects

   (b) Insects as mechanical vectors

   (c) Plant-insect coevolution

   (d) Genital appendages

4. Give an account on the structure of wings in insects. 6

   *Or*

   Write a brief note on different mouthparts of insects with respect to their feeding habitats.

5. Explain the digestive or endocrine system of insects with suitable labelled diagrams. 7





6. Discuss in detail about the social organization and behaviour in any one insect. 4+4=8

7. What is metamorphosis in insects? Give an account on insect hormones responsible for growth and development. 2+7=9

   *Or*

   Write short notes on any *two* of the following : 4½×2=9

   (a) Abdominal appendages

   (b) Insects as plant pest

   (c) Morphology of insect head

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Total No. of Printed Pages—3

**5 SEM TDC ZOOH (CBCS) C 11**

**2023**

( November )

ZOOLOGY

( Core )

Paper : C–11

**( Molecular Biology )**

*Full Marks* : 53
*Pass Marks* : 21

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

1. Fill in the blanks : 1×5=5

(a) Semiconservative replication of DNA was first demonstrated in _____.

(b) The process of synthesis of RNA from DNA template is called _____.

(c) Eukaryotic cells contain _____ distinct nuclear RNA polymerases that transcribe different classes of genes.

(d) The enzyme involved in amino acid activation is ____.

(e) DNA glycosylase is an enzyme involved in base excision repair. Its function is ____.

**2.** Write briefly about the following (any *two*) : 4×2=8

(a) Salient features of DNA and RNA

(b) Semiconservative nature of DNA replication

(c) Split gene

**3.** Explain the following (any *two*) : 4×2=8

(a) RNA interference

(b) Features of genetic code

(c) Watson and Crick model of DNA

**4.** List the enzymes involved in the process of DNA replication. Mention their functions. Explain the process of synthesis of lagging strand during DNA replication using suitable illustrations. 2+2+4=8

*Or*

Briefly explain the bidirectional nature of DNA replication. Give a note on DNA repair mechanism. 4+4=8





**5.** Explain the process of transcription in prokaryotes using suitable illustrations. 6+2=8

*Or*

Explain the formation of closed and open complex during the initiation of transcription. List the various transcription factors in prokaryotes and eukaryotes and mention their functions. 2+2+2+2=8

**6.** Explain the process of translation in prokaryotes using suitable illustrations. 8

*Or*

State the Wobble hypothesis. Describe the stage of initiation of translation in prokaryotes with appropriate illustrations. List the various initiation factors (IFs) involved and mention their functions. 2+4+2=8

**7.** What is RNA editing? Explain the process of RNA editing of the apolipoprotein B gene. 2+6=8

*Or*

What are post-transcriptional modifications? Explain the various post-translational modifications. 2+6=8

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Total No. of Printed Pages—3

## 5 SEM TDC ZOOH (CBCS) C 12

**2023**

( November )

ZOOLOGY

( Core )

Paper : C–12

**( Principles of Genetics )**

*Full Marks* : 53

*Pass Marks* : 21

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Fill in the blanks : 1×5=5

(a) 'One gene one enzyme' hypothesis was proposed by ______.

(b) Sigma factor is associated with sensitivity to ______.

(c) C-locus, which is responsible for colour in maize, is present on chromosome number ______.

Total No. of Printed Pages—16

**5 SEM TDC DSE PHY (CBCS)**
**2 (H) A/B/C**

**2023**

( November )

PHYSICS

( Discipline Specific Elective )

( For Honours )

Paper : DSE–2

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

Paper : DSE–2 (A)

**( Astronomy and Astrophysics )**

*Full Marks* : 80
*Pass Marks* : 32

**1.** Choose the correct answer from the following : 1×8=8

*(a)* The relation between parsec (pc) and astronomical unit (AU) is

*(i)* 1 Mpc = 206265 AU

*(ii)* 1 Mpc = $206265 \times 10^6$ AU

*(iii)* 1 pc = 2062·65 AU

*(iv)* 1 kpc = 206265 AU

(b) The luminosity of a star is related to its effective surface temperature as

(i) $L \propto T_{\text{eff}}$

(ii) $L \propto T_{\text{eff}}^2$

(iii) $L \propto T_{\text{eff}}^4$

(iv) $L \propto T_{\text{eff}}^6$

(c) On the celestial sphere, the north pole has a declination of

(i) $\pi$

(ii) $+\frac{\pi}{2}$

(iii) $-\frac{\pi}{2}$

(iv) 0

(d) The mass-luminosity relation is given by

(i) $L \propto M$

(ii) $L \propto M^2$

(iii) $L \propto M^{\frac{1}{2}}$

(iv) $L \propto M^3$





(e) Newtonian telescope used in astronomy is a

(i) reflecting telescope

(ii) refracting telescope

(iii) dispersive telescope

(iv) None of the above

(f) From the solar system, the galactic centre is situated at a distance of about

(i) 1 Mpc

(ii) 8 kpc

(iii) 1 AU

(iv) 1 kpc

(g) Andromeda Galaxy belongs to a galaxy of type

(i) elliptical

(ii) circular

(iii) spiral

(iv) irregular

(h) According to Harvard spectral classification, with decreasing temperature towards the right, the sequence is

(i) A–B–F–G–K–M–O

(ii) O–B–A–F–G–K–M

(iii) O–B–A–K–M–F–G

(iv) B–A–F–G–O–K–M

2\. *(a)* Define apparent and absolute magnitudes of a star and derive an expression for the distance modulus. 2+4=6

*Or*

Describe the trigonometric parallax method for measuring distance of stars. Can the distances of all stars be measured by this method? Why? 4+1+1=6

*(b)* The apparent magnitude of the sun is –26·8. Determine its absolute magnitude. It is given that the distance between the sun and the earth is $1 \cdot 5 \times 10^{11}$ m. 3

*Or*

Define luminosity and the radiant flux. 1½+1½=3

*(c)* Explain the direct method of determining the radii of stars. 3

*Or*

The distance modulus of the star Vega is –0·5. What is its distance from us?

*(d)* Using Stefan-Boltzmann law of radiation, obtain the ratio of radii $R_1$ and $R_2$ of two stars with surface temperatures $T_1$ and $T_2$ and of absolute magnitudes $M_1$ and $M_2$, respectively. 5





*Or*

The α Centauri binary system is 1·338 pc distant with a period of 79·92 years. The *A* and *B* components have a mean separation of 23·7 AU. What is the total mass of the system? If primary component, α Centauri *A* has a mean distance of 11·2 AU from the system's barycenter, what is the mass of each of the component stars in the system?

*(e)* Draw the celestial sphere showing the celestial poles, celestial equator, ecliptic, vernal equinox and autumnal equinox. 5

3\. *(a)* Explain the horizon coordinate system with proper diagram. Discuss the shortcomings of this system. 4+2=6

*(b)* What are light gathering power, resolving power and diffraction limit of an optical telescope? Calculate the diffraction limit of resolution of a 3 m telescope for the wavelength of 600 nm. 2+2+2+2=8

4\. *(a)* Draw a schematic diagram showing the layers of solar atmosphere. 4

(b) Write a short note on any *one* of the following : 3

(i) Solar activity

(ii) Extra solar planet

5. (a) Draw a labelled *H-R* diagram and explain its significant features. 3+2=5

(b) Draw blackbody radiation curves for three objects with mean temperatures $T_1$, $T_2$ and $T_3$ respectively, such that $T_1 > T_2 > T_3$. 3

6. Answer any *three* questions from the following : 5×3=15

(a) Make a sketch of the Milky Way Galaxy and label its various components. Show the position of the sun in it. 4+1=5

(b) With an appropriate diagram, explain Hubble's classification of galaxies. What is the type assigned to the Milky Way Galaxy? 4+1=5

(c) Distinguish between spiral and elliptical galaxies giving one example of each type. 5

(d) Draw the galaxy rotation curve. Describe how it can give an evidence of presence of dark matter. 2+3=5

( Continued )





7. (a) State Hubble's law. Given that the value of the Hubble constant is $70\ \text{kms}^{-1}\ \text{Mpc}^{-1}$, estimate the age of the universe. 1+5=6

*Or*

(b) Discuss the concept of cosmic ladder. Explain how Cepheid variables stars have been used for measuring distances of nearby galaxies. 2+4=6

Paper : DSE–2 (B)

**( Physics of Devices and Instruments )**

*Full Marks* : 53

*Pass Marks* : 21

1. Choose the correct answer from the following : 1×5=5

(a) A unijunction transistor has

(i) two *p-n* junctions

(ii) one *p-n* junction

(iii) three *p-n* junctions

(iv) None of the above



( Turn Over )

(b) In a P-channel JFET, the charge carriers are

(i) electrons

(ii) holes

(iii) both electrons and holes

(iv) protons

(c) Which of the following filter circuits results in the best voltage regulation?

(i) Chock input

(ii) Capacitor input

(iii) Resistance input

(iv) None of the above

(d) A Class 10 clean room can have

(i) 35 dust particles per $m^3$

(ii) 350 dust particles per $m^3$

(iii) 3500 dust particles per $m^3$

(iv) 10 dust particles per $m^3$

(e) Electronic-Grade Silicon (EGS) is

(i) polycrystalline material

(ii) single-crystal material

(iii) amorphous material

(iv) monocrystalline material





**2.** Discuss the construction and working of a unijunction transistor (UJT). Name some of its important applications. 3+1=4

*Or*

Discuss the construction and working of a JFET. Mention its advantages. 4

**3.** Write short notes on any *two* of the following : 3×2=6

(a) Depletion-mode MOSFETS

(b) MOS device

(c) Tunnel diode

**4.** (a) What is the need of filter circuit? Describe qualitatively the action of L filter. 1+3=4

*Or*

Give a comparison between active and passive filters with examples of each type. 4

(b) Explain the working of an astable multivibrator with a circuit diagram. 4

**5.** The intrinsic stand off ratio for a UJT is determined to be 0·6. If the inter-base resistance is 5 kΩ, what are the values of $RB_1$ and $RB_2$? 2

6. Draw the basic Phase-locked Loop (PLL) configuration. What are the purposes of loop filter in PLL? 2+2=4

7. (a) Why is wet etching process anisotropic? Mention the challenges related to etching process. 1+2=3

*Or*

Define the three parameters, which determine the performance of a lithographic exposure. 3

(b) Discuss different types of layering steps. 3

*Or*

Discuss briefly chemical vapour deposition method with its advantages.

8. Write notes on any *two* of the following : 2×2=4

(a) Photoresists

(b) Projection printing

(c) Shadow printing

(d) Electron lithography

9. Write the full form of RS-232. Which wires do the symbols 'Tx' and 'Rx' mean? 1+1=2





10. What are the features/advantages of GPIB? 2

11. Draw the circuit diagram of CE amplitude modulator. 2

12. What do you mean by amplitude modulation? Mention its different types and discuss briefly any one of it. 1+2+2=5

13. What is modulation index? Give its physical significance. 1+2=3

Paper : DSE–2 (C)

**( Physics of Earth )**

*Full Marks* : 80

*Pass Marks* : 32

1. Choose the correct answer/Fill in the blank from the following (any *eight*) : 1×8=8

(a) The solar system is located _____ of the Milky Way Galaxy.

(i) at the centre

(ii) in a spiral arm

(iii) at elliptic plane

(iv) at the edge

*(b)* The ice sheets of Greenland and Antarctica, continental glaciers and snowfields, sea ice and permafrost form the

*(i)* biosphere
*(ii)* geosphere
*(iii)* hydrosphere
*(iv)* cryosphere

*(c)* Volcanism is the process happening in the

*(i)* geosphere
*(ii)* atmosphere
*(iii)* biosphere
*(iv)* cryosphere

*(d)* In the stratosphere, the temperature increases with height due to the presence of

*(i)* ozone layer
*(ii)* ozone hole
*(iii)* less amount of gases
*(iv)* sulfuric acid droplets

*(e)* The frequency of the cosmic microwave background radiation falls in the ____ range of the electromagnetic spectrum.

*(i)* visible
*(ii)* infrared
*(iii)* microwave
*(iv)* radio frequency





*(f)* Earthquakes can occur with ____ faulting.

*(i)* normal
*(ii)* reverse
*(iii)* thrust
*(iv)* All of the above

*(g)* Which of the following describes the build-up and release of stress during an earthquake?

*(i)* Modified Mercalli scale
*(ii)* Principle of superposition
*(iii)* Elastic-rebound theory
*(iv)* Travel time difference

*(h)* The point where movement occurred and which triggered the earthquake is the

*(i)* dip
*(ii)* epicenter
*(iii)* focus
*(iv)* strike

*(i)* The present-day climate change is due to the

*(i)* change in earth's orbital parameters
*(ii)* solar activity
*(iii)* human activity
*(iv)* ozone hole

(j) The global warming is due to the

(i) natural greenhouse effect

(ii) enhanced greenhouse effect

(iii) runaway greenhouse effect

(iv) climate change

2. Answer the following questions :

(a) What is a meteoroid? State the difference between a meteor and a meteorite. 1+2=3

(b) State the differences between the terrestrial and Jovian planets. Why are they so named? 3+1=4

(c) Which planetary bodies in the solar system exhibit resemblance with the earth? Explain. 4

(d) What is cosmic microwave background radiation? Explain its relation with the origin of the universe. 1+4=5

*Or*

Discuss the different stages of the formation of a star. 5

3. Answer the following questions :

(a) Discuss the interaction between the different spheres and the solid earth. 5





(b) Explain how the glacier ice sheets and polar ice caps influence the earth's ecosystem. 3

(c) Name different layers of the earth's atmosphere. Discuss the variation of temperature with altitude. What constitutes the earth's atmosphere? 2+3+2=7

(d) Discuss the variation of pressure and density of the earth's atmosphere with altitude. 5

*Or*

Explain the components of the biosphere.

4. Answer the following questions :

(a) Distinguish between earthquake magnitude and intensity. Discuss various earthquake magnitude scales. 1+3=4

(b) What are faults? Discuss the different types of faults responsible for causing earthquakes. 1+2=3

(c) What are seismic waves? Discuss the different types of seismic waves and their characteristics that distinguish one from the other. 1+4=5

*Or*

(d) What are earthquakes? Discuss the different types of earthquakes based on their mode of origin. 1+4=5

**5.** Answer the following questions :

(a) Discuss the timeline of major geological and biological events of the earth. 1+4=5

*Or*

Discuss the role of the biosphere in shaping the environment. 5

(b) Discuss the future evolution of the earth and the solar system. 5

**6.** Answer any *two* of the following questions : 7×2=14

(a) Explain the role of Milankovitch cycle on paleoclimate change. Is this affecting the present-day climate change? Explain.

(b) Explain how human activity is affecting the environment.

(c) Explain water cycle. How does it help in maintaining a steady state?

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# 5 SEM TDC DSE PHY (CBCS) 1 (H)

**2023**

( November )

PHYSICS

( Discipline Specific Elective )

( For Honours )

Paper : DSE–1

**( Classical Dynamics )**

*Full Marks* : 80

*Pass Marks* : 32

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Choose the correct answer : 1×8=8

(a) The ratio between the electric field and the magnetic field is

(i) $\mu_0 \varepsilon_0$

(ii) $\dfrac{1}{\mu_0 \varepsilon_0}$

(iii) $\dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(iv) $\sqrt{\mu_0 \varepsilon_0}$

(b) A cylinder constrained to move on a plane such that its axis of symmetry is always parallel to the plane, then the degrees of freedom are

(i) 2

(ii) 4

(iii) 5

(iv) 6

(c) In variational principle, the line integral of some functions between two end points is

(i) zero

(ii) infinite

(iii) extremum

(iv) one

(d) For a particle moving under the action of conservative force, the Lagrangian of the system

(i) is independent of position

(ii) increases in the direction of conservative force

(iii) decreases in the direction of conservative force

(iv) More information needed



( Continued )



(e) A stick of one metre in length is moving away from an observer at a speed of 0·80*c*. The observer will see the length of the stick as

(i) 0·6 m

(ii) 1·255 cm

(iii) 1 m

(iv) 1·66 m

(f) A square is travelling with the velocity of light in its diagonal direction. The observer in the rest observes it as

(i) square

(ii) rectangle

(iii) parallelogram

(iv) rhombus

(g) In a nuclear plant, $10^{17}$ joule energy is available from mass conservation. How much mass was lost?

(i) 0·1 kg

(ii) 1 kg

(iii) 10 kg

(iv) 100 kg

(h) A fluid is called turbulent when

(i) the viscosity of fluid is high

(ii) Reynolds' number is greater than 2000

(iii) Reynolds' number is less than 2000

(iv) the density of fluid is low



( Turn Over )

2. (a) If a charged particle is moving through a transverse uniform electric field, show that the path of the particle will be parabola inside this field. 2

(b) State and prove Hamilton's principle. 3

*Or*

A particle of mass $m$ falls a given distance $z_0$ in time $t_0 = \left(\frac{2z_0}{g}\right)^{1/2}$ and the distance travelled in time $t$ is given by $z = at + bt^2$, where constants $a$ and $b$ are such that the time $t_0$ is always the same. Show that the integral $\int_0^{t_0} L\, dt$ is extremum for real values of the coefficients only when $a = 0$ and $b = \frac{g}{2}$.

(c) A particle of mass $m$ is projected with initial velocity $u$ at an angle $\alpha$ with the horizontal. Use Lagrange's equations to describe the motion of the projectile. The resistance of the air may be neglected. 3

3. (a) Establish the Euler-Lagrange equations of motion by differential method. 5

(b) Using Hamilton's equation of motion, deduce the equations of motion of a compound pendulum. 3





*Or*

Derive the Hamiltonian for a charged particle in electromagnetic field.

(c) Define generalized momentum. Establish the principle of conservation of angular momentum using generalized notations. 1+4=5

*Or*

An inverted pendulum consists of a particle of mass $m$ supported by a rigid massless rod of length $l$. The pivot $O$ has a vertical motion given by $z = A \sin \omega t$. Obtain the Lagrangian and differential equation of the motion. 5

4. (a) If a particle is slightly displaced from the point of equilibrium executing small oscillations, then calculate the potential energy about a point of stable equilibrium. 3

(b) Three rigid spheres are connected by light flexible rods with relative masses $m_1 : m_2 : m_3 = 1:2:1$. Describe all the normal modes of the system and the normal frequencies. 6

*Or*

Show that the total energy of a coupled system with three degrees of freedom is equal to the sum of energies of its principal mode of oscillation.

5. (a) State the postulates of special theory of relativity. 2

(b) Write short notes on any *two* of the following : 3×2=6

(i) Length contraction

(ii) Twin paradox

(iii) Momentum 4-vector

*Or*

An astronaut wishes to determine his velocity of approach as he nears the moon. He sends a radio signal of frequency $5 \times 10^9$ Hz and compares this frequency with its echo, observing a difference of 86 kHz. What is the velocity of space vehicle relative to the moon? (Use the terms of first-order $v/c$) 6

(c) Discuss space like, time like and light like intervals. 3

6. (a) Derive the expression of relativistic total energy. 4

(b) Explain the concept of simultaneity in the context of special relativity. 4

*Or*

A muon is travelling with speed $v = 0 \cdot 99c$ ($c$ stands for velocity of light) vertically down through the atmosphere. Its half-life in its own rest frame is 1·5 microsecond. What is its half-life as measured by an observer on the earth?





(c) Show that $E^2 - p^2c^2$ is invariant under Lorentz transformation. 4

7. (a) Define relativistic Doppler effect. Discuss the salient features of relativistic Doppler effect. 2+3=5

*Or*

Excited $Fe^{57}$ nuclei sometimes decay to produce γ-ray photon of frequency $3 \cdot 46 \times 10^{18}$ Hz. Find the frequency of the photon emitted at an angle of 60° in the laboratory frame relative to the direction of the $Fe^{57}$ nucleus, when it is moving with a velocity $6 \times 10^7$ ms$^{-1}$. 5

(b) Explain the concept of four-force and discuss the conservation of four-momentum. 2+3=5

*Or*

A beam of $10^4 \pi^+$ mesons moves in a circular path of radius 20 metre at a speed 0·99$c$. The proper mean life of the $\pi^+$ meson is $2 \cdot 5 \times 10^{-8}$ sec.

(i) How many mesons survive when the beam returns to the point of origin?

(ii) How many mesons would left in a beam that had remained at rest at the origin for the same period of time? 2+3=5

**8.** *(a)* Write the equation of continuity of flow of liquid and prove it. 2

*(b)* Write down Poiseuille's equation for liquid flowing through a capillary tube. From this equation, show that if two capillaries of radii $r_1$ and $r_2$ having lengths $l_1$ and $l_2$ respectively are set in series, the rate of flow $V$ is given by

$$V = \frac{\pi P}{8\eta}\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)^{-1}$$

where $P$ is pressure difference across the arrangement and $\eta$ is the coefficient of viscosity. 1+3=4

*Or*

Three capillaries of lengths $8L$, $0 \cdot 2L$ and $2L$, with radii $r$, $0 \cdot 2r$ and $0 \cdot 5r$ respectively are connected in series. If the total pressure across the system in an experiment is $P$, then calculate the pressure across the shortest capillary. 4

*(c)* Write the qualitative description of turbulence of a liquid. What is Reynolds' number? 1+2=3

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